EEL 4213 Power System I
Test # 2

1. At a large generation station, two generators are connected to the station bus, having the following parameters:

Generator 1	Generator 2
$X_d'' = 0.08\,pu$	$X_d'' = 0.09\,pu$
$X_d' = 0.12\,pu$	$X_d' = 0.13\,pu$
$X_d = 1.10\,pu$	$X_d = 1.20\,pu$
$\tau_d'' = 0.020$ s	$\tau_d'' = 0.020$ s
$\tau_d' = 0.500$ s	$\tau_d' = 0.500$ s

Generator 1 supplies 0.6 *pu* power at unity power factor and Generator 2 supplies 0.8 *pu* power at unity power factor to the transmission network. The station bus is maintained at 1.0 *pu* voltage by adjusting the excitation at each generator.

a) Calculate the excitation voltage behind each generator's synchronous reactance under normal operating conditions. (15 pts)

b) Using the excitation voltages of part a), determine the steady state, transient, and subtransient short circuit currents for a fault on the station bus. (15 pts)

c) The station's circuit breakers take 9.0 cycles (0.150 s) to open during a fault. Specify the maximum fault-interrupting current rating for the circuit break connecting the generator to the station bus. (10 pts)

a) $E_1 = 1.0\angle 0^\circ + j1.10\left(\frac{0.6 - j0.0}{1.0}\right) = 1.2\angle 33.42^\circ$

$E_2 = 1.0\angle 0^\circ + j1.20\left(\frac{0.8 - j0.0}{1.0}\right) = 1.39\angle 43.83^\circ$

b) $I_{d1}' = \frac{1.2}{j0.12} = 10$ $I_{d2}' = \frac{1.39}{j0.13} = 10.7$

$I_{d1}'' = \frac{1.2}{j0.08} = 15$ $I_{d2}'' = \frac{1.39}{j0.09} = 15.4$

$I_{d1} = \frac{1.2}{j1.10} = 1.09$ $I_{d2} = \frac{1.39}{j1.2} = 1.158$

c) $I'_{MAX\,GEN\,CB} = 10.7$

Problem Score:____________

2. Consider the following bus-impedance matrix that has been partially completed up to three nodal buses. Compute the new matrix for each new line (branch segment) listed below.

$$Z_{bus} = \begin{bmatrix} j0.08 & j0.03 & j0.07 \\ j0.03 & j0.05 & j0.04 \\ j0.07 & j0.04 & j0.11 \end{bmatrix}$$

a) Start from the given matrix above, add a branch between bus 2 and 3 having an impedance of j 0.05 pu (5 pts)

$$j\begin{bmatrix} 0.08 & 0.03 & 0.07 & -0.04 \\ & 0.05 & 0.04 & 0.01 \\ & & 0.11 & -0.07 \\ & & & 0.13 \end{bmatrix} \underset{\text{REDUC.}}{\overset{\text{KRON}}{\Rightarrow}} j\begin{bmatrix} 0.0677 & 0.0331 & 0.0485 \\ & 0.0492 & 0.0454 \\ & & 0.0723 \end{bmatrix}$$

b) Start from the given matrix above, add a branch starting from bus 2 to a new bus 4 having an impedance of j 0.03 pu (5 pts)

$$j\begin{bmatrix} 0.08 & 0.03 & 0.07 & 0.03 \\ & 0.05 & 0.04 & 0.05 \\ & & 0.11 & 0.04 \\ & & & 0.08 \end{bmatrix}$$

c) Start from the given matrix above, add a branch between bus 1 and the reference node having an impedance of -j 0.02 pu (5 pts)

$$j\begin{bmatrix} 0.08 & 0.03 & 0.07 & 0.08 \\ & 0.05 & 0.04 & 0.03 \\ & & 0.11 & 0.07 \\ & & & 0.06 \end{bmatrix} \underset{\text{REDUC.}}{\overset{\text{KRON}}{\Rightarrow}} j\begin{bmatrix} -0.0267 & -0.01 & -0.233 \\ & 0.035 & 0.005 \\ & & 0.0283 \end{bmatrix}$$

d) Start from the given matrix above, add a branch starting from the reference node to a new bus 4 having an impedance of j 0.05 pu (5 pts)

$$j\begin{bmatrix} 0.08 & 0.03 & 0.07 & 0 \\ & 0.05 & 0.04 & 0 \\ & & 0.11 & 0 \\ & & & 0.05 \end{bmatrix}$$

Problem Score:____________

3. A large network has the following bus-impedance matrices. Calculate the fault current, I_f and the resulting phase voltages (for each phase, V_a, V_b, & V_c,) of all four buses for the following faults. Assume a 1.0 per unit prefault voltage profile throughout the network.

$$Z^+_{bus} = Z^-_{bus} = j\begin{bmatrix} 0.05 & 0.01 & 0.03 & 0.02 \\ 0.01 & 0.06 & 0.04 & 0.03 \\ 0.03 & 0.04 & 0.05 & 0.02 \\ 0.02 & 0.03 & 0.02 & 0.05 \end{bmatrix} \qquad Z^0_{bus} = j\begin{bmatrix} 0.10 & 0.06 & 0.04 & 0.06 \\ 0.06 & 0.07 & 0.01 & 0.01 \\ 0.04 & 0.01 & 0.03 & 0.01 \\ 0.06 & 0.01 & 0.01 & 0.10 \end{bmatrix}$$

a) A single-line-to-ground fault at Bus 4 (10 pts)
b) A double-line-to-ground fault at Bus 3 (10 pts)
c) A line-to-line fault at Bus 2 (10 pts)
d) A three-phase fault at Bus 1 (10 pts)

a) $I_0 = I_1 = I_2 = \dfrac{V_4}{Z^0_{44} + Z^+_{44} + Z^-_{44}} = \dfrac{1.0}{j0.1 + 2\times j0.05} = -j5 = I_4^+ = I_4^- = I_4$

$I_f = 3I_0 = -j15$

$V_N^{[0+-]} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} Z^0_{N4} \cdot I^0_4 \\ Z^+_{N4} \cdot I^+_4 \\ Z^-_{N4} \cdot I^-_4 \end{bmatrix}$

$Z^0_{N4} \cdot I^0_4 = j\begin{bmatrix} .06 \\ .01 \\ .01 \\ .10 \end{bmatrix}(-j5) = \begin{bmatrix} 0.3 \\ 0.05 \\ 0.05 \\ 0.5 \end{bmatrix}$

$Z^+_{N4} \cdot I^+_4 = Z^-_{N4} \cdot I^-_4 = j\begin{bmatrix} 0.02 \\ 0.03 \\ 0.02 \\ 0.05 \end{bmatrix}(-j5) = \begin{bmatrix} 0.1 \\ 0.15 \\ 0.1 \\ 0.25 \end{bmatrix}$

$V_N^{ABC} = [A] \cdot \begin{bmatrix} -0.3 & -0.05 & -0.05 & -0.5 \\ 0.9 & 0.85 & 0.9 & 0.75 \\ -0.1 & -0.15 & -0.1 & -0.25 \end{bmatrix} = \begin{matrix} A \\ B \\ C \end{matrix}\begin{bmatrix} 0.5\angle 0 & 0.65\angle 0 & 0.75\angle 0 & 0\angle 0 \\ 1.11\angle -128 & 0.95\angle -114 & 0.98\angle -117 & 1.15\angle - \\ 1.11\angle 128 & 0.95\angle 114 & 0.98\angle 117 & 1.15\angle 1 \end{bmatrix}$

$N=1 \quad N=2 \quad N=3 \quad N=4$

b) $I_1 = \dfrac{V_3}{Z^+_{33} + \dfrac{Z^-_{33} Z^0_{33}}{Z^-_{33} + Z^0_{33}}} = \dfrac{1.0}{j0.05 + \dfrac{(j0.05)(j0.03)}{(j0.05)+(j0.03)}} = -j14.55 = I_3^+$

$I_2 = -\dfrac{V_3 - Z^+_{33} I^+_3}{Z^-_{33}} = -\dfrac{1-(j0.05)(-j14.55)}{j0.05} = j5.45 = I_3^-$

$I_0 = -\dfrac{V_3 - Z^+_{33} I^+_3}{Z^0_{33}} = -\dfrac{1-(j0.05)(-j14.55)}{j0.03} = j9.10 = I_3^0$

$I_f = 3I_3^0 = j27.3$

Problem Score:___________

Continuation of problem 3

b) CONT

$$Z_{N3}^{+} \cdot I_3^{+} = \begin{bmatrix} 0.4365 \\ 0.5820 \\ 0.7275 \\ 0.2910 \end{bmatrix} \quad Z_{N3}^{-} \cdot I_3^{-} = \begin{bmatrix} -0.1635 \\ -0.2180 \\ -0.2725 \\ -0.1090 \end{bmatrix} \quad Z_{N3}^{0} \cdot I_3^{0} = \begin{bmatrix} -0.364 \\ -0.091 \\ -0.273 \\ -0.091 \end{bmatrix}$$

$$V_N^{ABC} = [A] \cdot \begin{bmatrix} 0.364 & 0.091 & 0.273 & 0.091 \\ 0.5635 & 0.418 & 0.2725 & 0.709 \\ 0.1635 & 0.218 & 0.2725 & 0.109 \end{bmatrix} = \begin{bmatrix} 1.091\angle 0 & 0.727\angle 0 & 0.818\angle 0 & 0.909\angle 0 \\ 0.35\angle -89 & 0.29\angle -143 & 0\angle 0 & 0.61\angle -121 \\ 0.35\angle 89 & 0.29\angle 143 & 0\angle 0 & 0.61\angle 121 \end{bmatrix}$$

c) $I_0 = 0$

$$I_1 = -I_2 = \frac{V_2}{Z_{22}^{+} + Z_{22}^{-}} = \frac{1.0}{2(j0.06)} = -j8.3 = I_2^{+} = -I_2^{-}$$

$$I_f = -j\sqrt{3}\, I_1 = -14.38$$

$$Z_{N2}^{+} \cdot I_2^{+} = -Z_{N2}^{-} I_2^{-} = \begin{bmatrix} 0.083 \\ 0.498 \\ 0.332 \\ 0.249 \end{bmatrix}$$

$$V_N^{ABC} = [A] \cdot \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.917 & 0.502 & 0.668 & 0.751 \\ 0.083 & 0.498 & 0.332 & 0.249 \end{bmatrix} = \begin{bmatrix} 1.0\angle 0 & 1.0\angle 0 & 1.0\angle 0^\circ & 1.0\angle 0^\circ \\ 0.88\angle -124 & 0.5\angle -180 & 0.58\angle -150 & 0.66\angle -139 \\ 0.88\angle 124 & 0.5\angle 180 & 0.58\angle 150 & 0.66\angle 139 \end{bmatrix}$$

d)

$$I_f = \frac{V_1}{Z_{11}^{+}} = \frac{1.0}{j0.05} = -j20 = I_1^{+} \qquad I_1^{-} = I_1^{0} = 0$$

$$V_{N0}^{+} = 1 - Z_{N1}^{+} I_1^{+} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} - j \begin{bmatrix} 0.05 \\ 0.01 \\ 0.03 \\ 0.02 \end{bmatrix} (-j20) = \begin{bmatrix} 0 \\ 0.8 \\ 0.4 \\ 0.6 \end{bmatrix}$$

$$V_N^{ABC} = [A] \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.8 & 0.4 & 0.6 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0.8\angle 0 & 0.4\angle 0 & 0.6\angle 0 \\ 0 & 0.8\angle -120 & 0.4\angle -120 & 0.6\angle -120 \\ 0 & 0.8\angle 120 & 0.4\angle 120 & 0.6\angle 120 \end{bmatrix}$$

Problem Score:____________